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Standstill Electric Charge Generates Magnetostatic Field under Born-Infeld Electrodynamics

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Abstract The Abelian Born-Infeld classical non-linear Electrodynamic has been used to investigate the electric and magnetostatic fields generated by a point-like electric charge at rest in an inertial frame. The results show a rich internal structure for the charge. Analytical solutions have also been found. Such fields configurations have been interpreted in terms of vacuum polarization and magnetic-like charges produced by the very high strengths of the electric field considered. Apparently non-linearity is responsible for the emergence of an anomalous magnetostatic field suggesting a possible connection to that created by a magnetic dipole composed of two magnetic charges with opposite signs. Consistently in situations where the Born-Infeld field strength parameter is free to become infinite, Maxwell's regime takes over, the magnetic sector vanishes and the electric field assumes a Coulomb behavior with no trace of a magnetic component. The connection to other monopole solutions, like Dirac's and 'tHooft's Poliakov's types are also discussed. Finally, some speculative remarks are presented in an attempt to explain such fields.

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1 Introduction

This work investigates, under a classical approach and exploring the non-linear properties of the Abelian Born-Infeld Theory (B-I), the configuration of the fields generated by a single electric charge at rest. The main motivation for this paper came from the question if a pure electric point-like charge at rest in an inertial frame generates some kind of magnetostatic field. The challenge lies on finding classical solutions for B-I magnetic sector without speculative additional assumptions. Only a few suitable restrictions have been imposed. The results can be listed as following below:

(a) There is analytical and real stable magnetostatic dipole-like solution generated by intense electric field strength;
(b) It is possible to define a single magnetic charge in terms of the electric point-like charge;
(c) The findings is consistent concerning to the Maxwell linear theory. In other words, it vanishes when the B-I parameter b is free to become infinite;
(d) The magnetic charge intensity calculated is close Dirac's prediction.

B-I non-linear classical Electrodynamics [1, 2, 8–10] represents an advanced theory to explain the structure and the finite energy of the electron. It emerges in the more broad context of the M-Theory, where the Superstrings Theories are enclosed. Recent revival of nonlinear electrodynamics has been verified, mainly due to the fact that these theories appear as effective theories at different levels of string/M-theory, in particular in Dp-branes and super-symmetric extensions, and non-Abelian generalizations. B-I Lagrangian describes the electromagnetic fields that live on the world-volume of D-branes and T-duality gives direct evidence that it governs the dynamics of the electromagnetic fields on D-branes [3–7]. B-I Lagrangian density is one of the general non-derivative Lagrangians which depend only on the two algebraic Maxwell invariants. Among others its most attractive properties B-I Lagrangian is one of the simplest non-polynomials that preserve gauge and Lorentz invariance, the vacuum is characterized by $f_{\mu\nu}=0$ and the energy density is positive. The field strength $f_{\mu\nu}$ is finite everywhere and is characterized by its length r_o. B-I Theory is the only non linear electrodynamic theory ensuring the absence of bi-refringence, this is, the vacuum light speed is always c.

The organization of this paper is as follows: in Sect. 2, one accounts for the exposition of the problem and the formulation of the main assumptions and constraints. In Sect. 3 one gets and solves the angular and radial differential equations. Section 4, sets up the connection between angular moment and magnetic charge. Section 5 fixes the single magnetic charge. Section 6 makes numerical calculation and Sect. 7 presents some Final Considerations.

2 Classical Born-Infeld Equations in Minkowski Space-Time

In these two sub-sections, one exposes the standard Abelian B-I theory embedded in flat space. The signature of the metric tensor and the first assumption is established. The constitutive relations are suitably built up in order to make sure the integrability of the system.

2.1 The Electric Charge at Rest

The B-I non-linear Electrodynamics action [1] is defined, in Minkowski space-time, as:

$$S = \int d^4 x b^2 \left[1 - \sqrt{-\det\left(\eta_{\mu\nu} + \frac{f_{\mu\nu}}{b} \right)} \right].$$

The metric tensor $\eta_{\mu\nu}$ has signature $(1, -1, -1, -1)$ and $f_{\mu\nu}$ is the electromagnetic tensor. The parameter b, like the speed of light in Einstein's relativity theory, is the maximum field strength allowed by the B-I Theory and has a large value (about 10^{15} esu). Setting its value to infinity, leads to Maxwell's linear Electrodynamics. That means that there is no limit to the field strength in Maxwell linear Electrodynamics. Enclosed in the action integral is the Born-Infeld Lagrangian and evaluating its determinant yields:

$$L = b^2 \left[1 - \sqrt{1 - \frac{E^2 - B^2}{b^2} - \left(\frac{\vec{E} \cdot \vec{B}}{b^2} \right)^2} \right].$$

In addition, the electric induction, $\vec{D}$, and the magnetic field, $\vec{H}$, as derived from the canonical relations, are:

$$\vec{D} = \frac{\partial L}{\partial \vec{E}} = \frac{\vec{E} + (\frac{\vec{E} \cdot \vec{B}}{b^2}) \vec{B}}{\sqrt{1 - \frac{E^2 - B^2}{b^2} - (\frac{\vec{E} \cdot \vec{B}}{b^2})^2}}, \tag{1}$$

$$\vec{H} = \frac{\partial L}{\partial \vec{B}} = \frac{\vec{B} - (\frac{\vec{E} \cdot \vec{B}}{b^2}) \vec{E}}{\sqrt{1 - \frac{E^2 - B^2}{b^2} - (\frac{\vec{E} \cdot \vec{B}}{b^2})^2}}. \tag{2}$$

The interaction with other charged particles is introduced by adding a term $j_\mu A^\mu$ to the B-I Lagrangian. The equations of motion are the standard Maxwell equations and the non-linearity is inserted in (1) and (2). For a static point-like charge these equations, on macroscopic level, are:

$$\vec{\nabla} \cdot \vec{D} = e\delta(\vec{x}), \qquad \vec{\nabla} \times \vec{E} = \vec{0},$$
$$\vec{\nabla} \cdot \vec{B} = 0, \qquad \vec{\nabla} \times \vec{H} = \vec{0},$$

and the solution for the electric induction $\vec{D}$, by taking $\vec{B} = \vec{H} = \vec{0}$, is well known [1, 2, 5]. It is singular and identical to the Maxwell solution, while the field $\vec{E}$ remains well defined at all points, even at $r = 0$. Thus:

$$\vec{D} = \frac{e}{4\pi r^2} \hat{r} \qquad \text{and} \qquad \vec{E} = \frac{e}{\sqrt{r^4 + r_o^4}} \hat{r},$$
$$r_o = \sqrt{\frac{e}{4\pi b}}, \quad \hat{r} = \vec{r} / |\vec{r}|. \tag{3}$$

These are the necessary tools, for the present work, to describe that electric charge in more broad way. No change will be done on electric sector and it will be preserved by appropriate assumptions raised further.

2.2 The Non-Trivial Magnetostatic Sector

What are the consequences of not setting $\vec{B}$ and $\vec{H}$ equal to zero? What kind of fields could the theory provide? If real and finite solutions do exist, then what originates those fields? In order to try to answer such questions, one must consider each component of the constitutive equations (1) and (2) at a time. By assuming that the magnetic sector has only radial and polar components, that are functions of the radial distance r from the point-like charge, and the polar angle θ, then the following relations for $\vec{B}$ and $\vec{H}$ come about:

$$\begin{aligned} \vec{H}(r,\theta) &= H_r(r,\theta)\widehat{r} + H_\theta(r,\theta)\widehat{\theta}, \\ \vec{B}(r,\theta) &= B_r(r,\theta)\widehat{r} + B_\theta(r,\theta)\widehat{\theta}, \end{aligned} \tag{4}$$

where the subscripts refer to the components of the vector. So, the problem is axially symmetric. The introduction of φ dependence will violate the $\vec{\nabla} \times \vec{H} = \vec{0}$.

Then, the constitutive relations (1) and (2) for each component is written leading to an algebraic non-linear system of equations, relating all electric and magnetic components. The condition $|E_\theta| \ll |\frac{B_r}{B_\theta} E_r|$ is necessary in order to get only radial component of electric sector.

$$E_r = D_r \frac{R}{\left(1 + \frac{B_r^2}{b^2}\right)}, \qquad E_\theta \approx 0, \tag{5}$$

$$B_r = H_r \frac{R}{\left(1 - \frac{E_r^2}{b^2}\right)}, \qquad B_\theta = H_\theta R, \tag{6}$$

with

$$R = \sqrt{\left(1 + \frac{B_r^2}{b^2}\right)\left(1 - \frac{E_r^2}{b^2}\right) + \frac{B_\theta^2}{b^2}}.$$

The magnetostatic field components (B_r and B_θ) are then required to satisfy the following constraints, $B_r \ll b$ and $B_\theta \ll b$ at all points, while preserving the major feature of the theory, that is, its nonlinearity. This is the first assumption introduced and it is necessary in order to ensure that the system remains integrable. The aforementioned set of equations is then reduced to:

$$R = \sqrt{\left(1 + \frac{B_r^2}{b^2}\right)\left(1 - \frac{E_r^2}{b^2}\right) + \frac{B_\theta^2}{b^2}} \approx \sqrt{1 - \frac{E_r^2(r)}{b^2}},$$

$$E_r(r) = \frac{D_r(r)}{\sqrt{1 + \frac{D_r^2(r)}{b^2}}}, \tag{7}$$

$$B_r(r,\theta) = H_r(r,\theta)\sqrt{1 + \frac{D_r^2(r)}{b^2}}, \tag{8}$$

$$B_\theta(r,\theta) = \frac{H_\theta(r,\theta)}{\sqrt{1 + \frac{D_r^2(r)}{b^2}}}. \tag{9}$$

That action keeps the B-I original behavior of the electric sector while still preserving the connection between the magnetic and electric sectors. At this point, one concludes that, under the first assumption, (7)–(9) tell that the electric sector shall not be affected by the induced magnetic sector. However, each one depends on $D_r(r)$, a subtle message from the electric to the magnetic sector.

3 Solution to the Born-Infeld Equation

This section is devoted to set up and solve the main differential equation. One more basic assumption is necessary in order to turn the system integrable.

The field $\overrightarrow{H}(r,\theta)$ is to satisfy $\overrightarrow{\nabla}\times\overrightarrow{H}(r,\theta)=\overrightarrow{0}$ and the field $\overrightarrow{B}(r,\theta)$ must be such that $\overrightarrow{\nabla}\cdot\overrightarrow{B}(r,\theta)=0$. We get two components PDE:

$$\frac{1}{r}[\partial_r(rH_\theta(r,\theta))-\partial_\theta H_r(r,\theta)]=0,$$

$$\frac{1}{r^2}\partial_r[r^2B_r(r,\theta)]+\frac{1}{r\sin(\theta)}\partial_\theta[\sin(\theta)B_\theta(r,\theta)]=0.$$

Now, we assume that the variables can be separable. This is the second assumption of our approach. The magnetic field components can be written as:

$$H_r(r,\theta)=h_r(r)G(\theta),\qquad H_\theta(r,\theta)=h_\theta(r)J(\theta). \tag{10}$$

The magnetic induction components are then determined by the magnetic field and become:

$$B_r(r,\theta)=b_r(r)G(\theta)=h_r(r)\sqrt{1+\frac{D_r^2(r)}{b^2}}G(\theta), \tag{11}$$

$$B_\theta(r,\theta)=b_\theta(r)J(\theta)=\frac{h_\theta(r)}{\sqrt{1+\frac{D_r^2(r)}{b^2}}}J(\theta), \tag{12}$$

where $h_r(r)$, $h_\theta(r)$, $G(\theta)$ and $J(\theta)$ are unknown functions to be determined and $b_r(r)=h_r(r)(1+\frac{D_r^2(r)}{b^2})^{1/2}$ as well $b_\theta(r)=h_\theta(r)(1+\frac{D_r^2(r)}{b^2})^{-1/2}$.

Then, substituting $H_r(r,\theta)$ and $H_\theta(r,\theta)$ on $\overrightarrow{\nabla}\times\overrightarrow{H}(r,\theta)=\overrightarrow{0}$, yields a set of two differential equations, where λ is a constant:

$$\frac{1}{h_r(r)}\frac{d[rh_\theta(r)]}{dr}=\frac{1}{J(\theta)}\frac{dG(\theta)}{d\theta}=\lambda. \tag{13}$$

Likewise, the substitution of $B_r(r,\theta)$ and $B_\theta(r,\theta)$ in $\overrightarrow{\nabla}\cdot\overrightarrow{B}(r,\theta)=0$ leads to:

$$\frac{1}{rb_\theta(r)}\frac{dr^2b_r(r)}{dr}=\frac{-1}{\sin(\theta)G(\theta)}\frac{d(\sin(\theta)J(\theta))}{d\theta}=\varsigma, \tag{14}$$

where ς is another constant. This can be collected in a system of four differential equations further.

$$\frac{d[rh_\theta(r)]}{dr}=\lambda h_r(r), \tag{15}$$

$$\frac{dG(\theta)}{d\theta}=\lambda J(\theta), \tag{16}$$

$$\frac{d}{dr}\left[r^2 b_r(r)\right] = \varsigma r b_\theta(r), \tag{17}$$

$$\frac{d(\sin(\theta)J(\theta))}{d\theta} = -\varsigma \sin(\theta)G(\theta). \tag{18}$$

Considering the angular equations (16) and (18), we arrive at a second order differential equation for $G(\theta)$.

$$\frac{d}{d\theta}\left\{\sin(\theta)\frac{d(G(\theta))}{d\theta}\right\} + \lambda\varsigma \sin(\theta)G(\theta) = 0, \tag{19}$$

which has the general angular solution given in terms of the Legendre functions of the first kind, $P_n(\cos(\theta))$, and Legendre functions of second kind, $Q_n(\cos(\theta))$. The second has the undesirable feature of not be always real and the angular solution can be written as:

$$G_n(\theta) = C_1 P_n(\cos(\theta)), \quad n = \frac{\sqrt{1+4\lambda\varsigma}}{2} - \frac{1}{2}.$$

By proceeding analogously, the result is a differential equation for $\Psi(r) = rh_\theta(r)$:

$$\frac{d}{dr}\left\{\sqrt{r^4 + r_o^4}\frac{d\Psi(r)}{dr}\right\} - \frac{\lambda\varsigma\Psi(r)r^2}{\sqrt{r^4+r_o^4}} = 0. \tag{20}$$

The general solution is the associated Legendre functions of the first and second kind $P_n^{1/4}(z)$ and $Q_n^{1/4}(z)$, with $z = \sqrt{(\frac{r}{r_o})^4 + 1}$ and n exactly the same for the angular sector. Once again, the second one takes complex values and the solution reduces to:

$$\Psi_n(r) = \overline{m}\sqrt{r}P_n^{\frac{1}{4}}(z), \quad z = \sqrt{\left(\frac{r}{r_o}\right)^4 + 1}. \tag{21}$$

Acceptable solutions to (19) require that n must be a natural number, otherwise the field lines will not be closed. As a result, $\lambda\varsigma$ assumes special integer values $(0, 2, 6, 12, \ldots)$. Looking for solutions (21), the component $h_\theta(r)$ is calculated and from (12) the component $b_\theta(r)$ are obtained. However, among this infinite set only two solutions will give physical meaning. For $\lambda\varsigma \geq 6$, the component $h_\theta(r)$ grows as the radial distance become large. For $\lambda\varsigma = 0$, there is no angular dependence, $P_0(\cos(\theta)) = 1$, and the field will be whole radial like a monopole field. In this situation, there is no angular moment and no way to express the magnetic charge in terms of electric charge. On the other hand the first assumption is violated whenever $r \approx 0$ because (17) tells that $b_r(r)$ grows up without limit. Taking $\lambda\varsigma = 2$, the situation is different, $G_1(\theta) = C_1\cos(\theta)$, and (16) gives the second angular function such that, $J_1(\theta) = -C_1\sin(\theta)/\lambda$. The solution (21) becomes [17]:

$$\Psi(r) = \overline{m}\sqrt{r}P_{\frac{1}{4}}^{\frac{1}{4}}(z) \longrightarrow h_\theta(r) = \frac{\overline{m}\sqrt{r}P_{\frac{1}{4}}^{\frac{1}{4}}(z)}{r},$$

and the constant $\overline{m}$ will be closely related to the magnetic dipole moment for suitable choice of the constants λ and ς. It is then possible to represent the magnetostatic field polar component, $b_\theta(r)$, like below:

$$b_\theta(r) = \frac{h_\theta(r)}{\sqrt{1+\frac{D_r^2(r)}{b^2}}}.$$

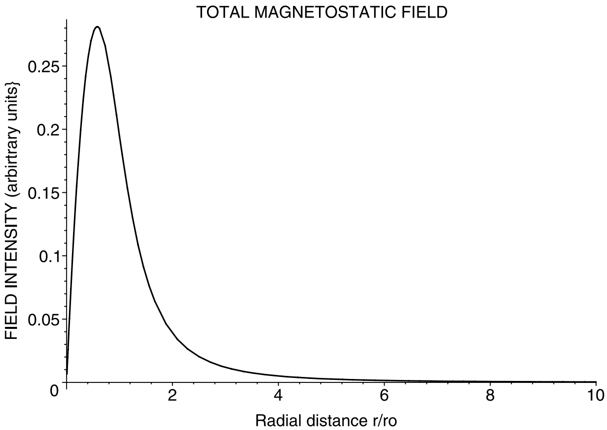

Fig. 1 Field as function of the charge distance

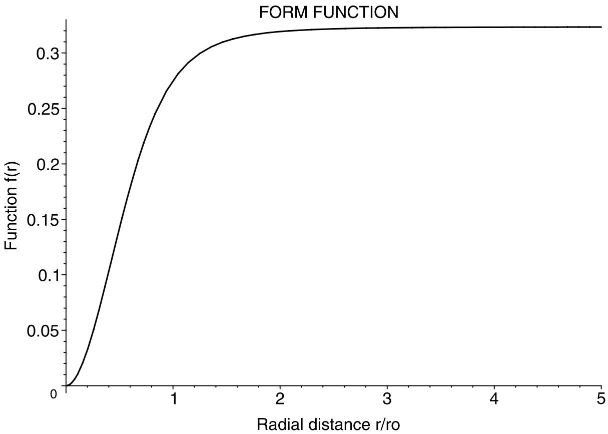

Fig. 2 Asymptotic behaviour of form function

The differential equation (17) allows the calculation of $b_r(r)$. Each component, polar and radial, is finite everywhere, and the Fig. 1 shows the strength of the total magnetostatic field, in arbitrary units, as a function of the radial distance of the electric point-like charge.

There is a maximum near the r_o and the field vanishes as $r \to 0$ or $r \to \infty$. An accurate mathematical analysis shows that the asymptotic behaviour is the well known r^{-3}, for $r \gg r_o$, indicating the connection of $\overline{m}$ with some magnetic dipole moment, and proportional to r for $r \approx 0$, indicating some induced magnetic charge distribution. Out of that range the components recover their linearity. To see this one takes $h_\theta(r)$ as:

$$\begin{aligned} h_\theta(r) &= \frac{\overline{m} r_o^2 f(z)}{r^3}, \\ f(z) &= z^2\left[\sqrt{z} P_{\frac{1}{4}}^{\frac{1}{4}}\left(\sqrt{1+z^4}\right) - kz\right]. \end{aligned} \tag{22}$$

The additional term kz is exact solution of (20) and $k = 0.8221789587$ is a constant that ensures $h_\theta(r)$ to vanish when $r \to \infty$. The function $f(z)$, called here as "*form function*" can be showed in Fig. 2. The asymptotic behavior is constant indicating that closely connection of $\overline{m}$ with the magnetic dipole moment.

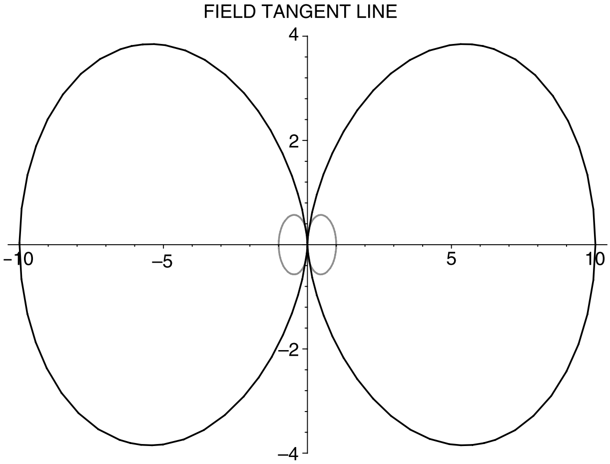

Fig. 3 Dipole line curve pattern

Tangent field lines can be evaluated by equating the ratio of field components to the slop of the curve in polar coordinates:

$$\frac{B_r(r,\theta)}{B_\theta(r,\theta)} = \frac{dr}{r d\theta}. \tag{23}$$

Integrating the last equation for two different asymptotic distances from the electric charge one arrives to the following equations in polar coordinates:

$$r = k_1 \sin^2(\theta), \quad r \gg r_o,$$
$$r = k_2 \sin^{2/3}(\theta), \quad r \approx 0.$$

Figure 3 shows the tangent closed lines for the total magnetic induction. It is a typical magnetic dipole pattern. The constants k_1 and k_2 are set up for different range of radial distance. For small loops, the entire curve is a good approximation. The other has some restrictions for polar angles near north and south poles.

The interpretation carried out is as if two magnetic charges of opposite signal distributed around the poles. The vacuum then polarized by the intense electric field behaves like a material surrounding the electric charge. This induced magnetic field may produce an angular momentum whenever it couples to an electric field.

4 The Angular Momentum and the Connection with the Magnetic Charge

In this section, the proposed induced magnetic charge, interpreted in last section, is written as a combination of fundamental parameters like the electric charge, e, and the B-I maximum field strength, b. In order to understand what this solution means, the angular momentum of those fields must be calculated due to its importance [11–13]. From the definition, $\overrightarrow{L}$ can be evaluated from the fields and its interpretation follows immediately:

$$\overrightarrow{L} = \int \overrightarrow{x} \times (\overrightarrow{D} \times \overrightarrow{B}) d^3\overrightarrow{x}. \tag{24}$$

Using the symmetry properties of the above integral yields the angular moment, that is completely aligned to the polar z axis:

$$L_z = \frac{\pi \overline{m} r_o \gamma}{2} e, \\ \gamma = \int_0^\infty dz \frac{f(z)}{\sqrt{1+z^4}} \approx 0.4003. \tag{25}$$

The result (25) suggests that an interpretation may be given in terms of a magnetic charge. It is known that the classical angular moment [13] for a system consisting of an electric monopole and a magnetic monopole is ge, the parameter g being the strength of the magnetic charge. If both results are compared, it is then possible to explain such magnetic charge as the result of a non-linearity effect or of the vacuum polarization of the B-I theory:

$$g_{eff} \to \frac{\pi \overline{m} r_o \gamma}{2}. \tag{26}$$

That is consistent with the rigorously linear behavior found whenever the maximum field strength is allowed to become infinite. In that case the contribution to the magnetic sector vanishes and that of the magnetic charge completely disappear. Based on the results presented, a proposed classical model for the electric charge structure could consist of an electric monopole, of strength e, and two distributed magnetic charge g, of opposite signs, in each hemisphere, yielding a total null magnetic charge when seen from a large distance, and null net divergence, like a magnet with its closed field lines.

5 Field as a Source of Field

Taking advantage of the result of the previous section and of the separation of variables, we speculate on the possibility of get a single magnetic charge. There is another way to interpret the result of the previous section. Looking for the divergence of the vector $\overrightarrow{B}(r,\theta)$ and removing the angular solution covering it, the term $-2\frac{b_\theta(r)}{r}$ can bee interpreted as a source of the pure radial field. That claims for a divergence of some radial field, $\overrightarrow{\nabla}_r \cdot \overrightarrow{B}_r$ with the transverse field component as a source of a pure radial field given by a spread magnetic density $\rho_m(r)$:

$$\frac{1}{r^2}\frac{d[r^2 b_r(r)]}{dr} + 2\frac{b_\theta(r)}{r} = 0, \tag{27}$$

$$\overrightarrow{\nabla} \cdot [b_r(r)\widehat{r}] = \rho_m(r), \quad \rho_m(r) = -2\frac{b_\theta(r)}{r}. \tag{28}$$

The total free magnetic charge strength, $2g$, is obtained by integrating $\rho_m(r)$ over the whole space:

$$2g = 2\int d^3\overrightarrow{x}\,\frac{b_\theta(r)}{r} = 8\pi\gamma\overline{m}r_o. \tag{29}$$

This result differs from (26) only in magnitude; it is larger than g_{eff}. Our interpretation is that the symmetry cancels all three directions contributions to the angular moment. When the symmetry is broken this cancellation despairs. Extracting it from the z component of the angular momentum, we actually compute only a fraction of the total. The remaining value is hidden by the symmetric components cancellation on interacting with the electric charge to produce the angular moment. Moreover the shadowing between both magnetic charges reduce the effective individual magnetic charge intensity given $g > g_{eff}$.

6 Numerical Calculations

It is interesting to know the intensity of the derived magnetic charge. For this, task in this section, we display the calculation of the magnetic charge recovering the SI units of the objects in (29). The function $\overline{m} r_o^2 f(z)$, here interpreted as the B-I magnetic dipole moment, can be given roughly as an effective charge e spinning around the center of a circle of radius r_o. With that we find that $\overline{m} \simeq \frac{ec}{2r_o}$, when the speed of that closed stead motion is took as $v \to \gamma c$. Restoring the SI units one has:

$$2g \to e\mu_o \gamma c.$$

We recall that the unique connection with B-I is the parameter $gamma$. The constant $\mu_0 = 4\pi \times 10^{-7}$ n/A^2 is the vacuum magnetic permeability and c is the speed of light. So, we have:

$$\frac{g}{e} \to \frac{\gamma \mu_o c}{2} = \frac{\gamma}{2}\sqrt{\frac{\mu_o}{\varepsilon_o}}.$$

The constant $\varepsilon_o = 8.854 \times 10^{-12}$ nm^2/C^2 is the electric permittivity of free space. The square root is the vacuum impedance of the vacuum. After the calculations, we arrive at a magnetic charge about $75.34e$. Dirac's prevision, based on Quantum Mechanics, gives a value equal to $68.5e$. In our picture of the magnetic dipole, we get an estimation compatible with the value found by Dirac.

Most of the results reported here are new and they do not simply result from numerical coincidences. They need however to be better understood. We ordinarily do not expect that quantum corrections can be responsible for a ten percent variation with respect to the value we have for the Dirac's monopole, used here as reference. Likewise, the separation of variables adopted when solving the equations should not be looked upon as the responsible for such a deviation either, since the equations implicitly bear that property. All the B-I contribution is encoded in the gamma parameter, which contains the term named "form function", and it is likely that a more precise knowledge of the numerical value for the maximum field could lead to a more accurate value for the magnetic charge. In view of that, some basic questions still remain to be answered: (a) Why does B-I non-linearity generate a magnetic charge strength close to the Dirac's monopole value? (b) Would there be any narrow and deep connection between the B-I classical configuration of the intense electric field in the vicinity of the electric charge and the Dirac's monopole derived from quantum mechanics?

7 Final Considerations

We showed that Classical Abelian Born-Infeld Electrodynamics can predict the existence of real and well-behaved magnetostatic fields solutions associated with electric charges at rest. Definitely, it is a non-linear effect simply ruled out by Maxwell's Electrodynamics. Although Born-Infeld non-linear Electrodynamics has not yet been experimentally confirmed, it was originally conceived to describe the electron properties based only on the structure of the field. The results of this work apparently suggest that the fields, rather than resembling Dirac's or t'Hooft's magnetic monopoles [15, 16], exhibit properties similar to those produced by a magnet as considered from the macroscopic point of view, although its more complex structure is only seen at the microscopic level. In addition, they indicate that the magnetostatic solutions ensued from the breaking of the radial symmetry. That was necessary in order to see the dependence of the magnetic charge on basic parameters such the maximum field strength b and the electric charge e. Such results seem to suggest the existence of vacuum excitation effects caused by the intense electric field strength around the electric charge. In addition by singling out the angular dependence, the current approach allows the investigation of a pure magnetic radial field generated by a spatially distributed magnetic charge derived from an electric charge. It must be stressed that quantum effects dominate over the classical description when distances in the order of 10^{-15} m are considered and the Compton wavelength of the electron, $\hbar/m_e c$, is about 10^{-12} m. Hence, r_o still lies within the limits of the classical validity, and a quantum Born-Infeld Theory still remains to be developed.

One of the major motivations for the use of the Born-Infeld electromagnetic field theory is to overcome the infinity problem associated with a point-like charge source as in Maxwell's theory. Born's original theory may currently be explained as an attempt to find classical solutions to represent electrically charged states produced by sources that have finite self-energy. We propose an extension to that theory by also seeking magnetically stable solutions derived for purely electrical charges. As expected, all additional anomalous magnetic terms vanish when Maxwell's regime is restored by allowing the maximum field strength to become infinite.

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